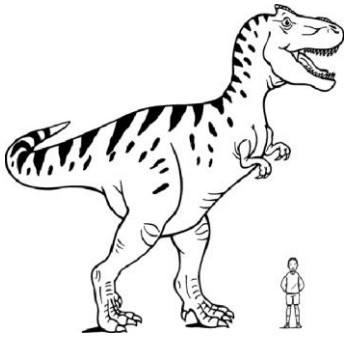
09. PROPORTIONALITY AND PERCENTAGES

DINOSAURS

You see a picture like this and you wonder how large the dinosaur was. You know the man is 1.80 m tall, could you deduce the side of the dinosaur, that is, its height and its length? What is picture scale?



HEALTHY FOOD

Nowadays it is very common to find a label in many packages of food.

It contains a variety of information about the nutritional value of the food item.

There are many pieces of information which are standard on most food labels.

This information helps people who are trying to restrict their intake of fat, sodium, sugar, or other ingredients, or those individuals who are trying to get enough of the healthy nutrients.

In the example we can see five topics: calories –the total energy that provides the product-; sugar, fat, sat fat –saturated fat- and salt.

And for each we can get three sort of information: percentage, that is, the proportional amount of this ingredient we get eating the indicated weight of the product. The total weight of this ingredient for the indicated amount too, and, finally, a classification of the product: low, medium and high, that tells us how the proportion of this substance in the product is.

Could you answer according to the information...? (You have to include your operations)

a) What is the healthy intake of calories a day?

b) And of sugar?

c) And of saturated fat?

d) And of salt?

1. RATIO AND PROPORTION

Division is the best operation to compare two quantities.

It tells me how a quantity is with respect to another.

For example, Spain has 40 millions residents and Portugal 20 millions; so Spain has double population than Portugal. $\frac{40}{20}=2$

Mary earns €300 a week in Movistar and Antonio €200; so Mary receives 1.5 times more salary than Antonio per week. $\frac{300}{200}=1.5$

RATIO

A ratio is a fraction of two quantities 'a' and 'b'.

That is a / b or a : b (read as 'a' to 'b').

It indicates how many times one quantity is greater or smaller than the other.

This concept is used to compare two like quantities –two lengths for instance- or two unlike quantities –one length and one time-.

Ex: $\frac{2m}{1m}=2$. The numerator is twice the denominator. Like quantities. The ratio doesn't have unit.

Ex: $\frac{100Km}{2hs}$. Unlike quantities. This ratio has unit. This is the speed of a motorbike.

Ex: A map scale is 1:150. What does it mean?

PROPORTION

Proportion is the equality of two ratios. That is $\frac{a}{b}=\frac{c}{d}$.

$\frac{100Km}{2hs}=\frac{200km}{4hs}$. This is a proportion. That is the ratios are proportional.

It could be two cars travelling at the same speed.

The result of the division is the constant of proportionality.

$\frac{100Km}{2hs}=\frac{200km}{4hs}=50km/h$.

The cars have the speed in common.

Fundamental property for a proportion

Product of extremes = Product of Means.

Quantity 1	a		c
Quantity 2	b		d
	$\frac{a}{b}=\frac{c}{d}$ or a · d = b · c		

2. DIRECT PROPORTIONALITY

MAGNITUDES IN DIRECT PROPORTION

Two magnitudes in a situation or experiment are in direct proportion if the quotient of every related values is constant.

That is, they form a proportion; the increase in one of them means an increase in the other.

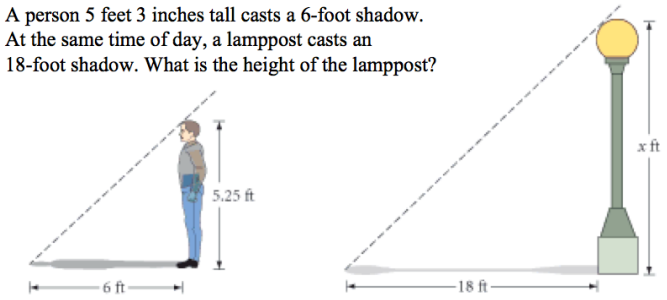
$$\frac{Magnitude\ 1}{Magnitude\ 2}=\text{constant}$$

The constant is the constant of proportionality.

Examples

The height and the shadow.

$$\frac{Height}{Shadow}=\frac{10m}{30m}=\frac{2m}{6m}=\text{____}$$



The weight and the price of something.

$$\frac{\Pr ice(€)}{Weight(kg)}=\frac{2€}{1kg}=\frac{4€}{2kg}=\text{____}$$

The price of a product and the tax to pay.

$$\frac{\Pr ice(€)}{Taxes(€)}=\frac{100€}{21€}=\frac{200€}{42€}=\text{____}$$

PROBLEMS OF DIRECT PROPORTIONALITY

We can solve these problems in two different ways:

Calculation of the proportionality constant

We calculate the value of a single unit.

Example: If 4 kg of potatoes cost 6€, what is the price of 9kg?

Rule of three direct

It is based on the fundamental property. Product of extremes = Product of Means.

Example: If we have to pay 21% of V.A.T. for a $250 item; how much tax we have to pay for it?

3. INVERSE PROPORTIONALITY

MAGNITUDES IN INVERSE PROPORTION

Two magnitudes are in inverse proportion if the product of every related values is constant, and finally, the increase of one of them is equal to the decrease of the other.

Magnitude 1 · magnitude 2 = constant

Examples

Number of workers and individual working time. $\frac{\text{Number of workers}}{\textit{Individual working time}} = \frac{8}{2} = \frac{4}{4} = \_\_\_\_$	
Number of participants buying a gift and their contribution. $\frac{\text{Number of participants}}{\textit{Individual contribution}} = \frac{10}{6} = \frac{5}{12} = \_\_\_\_$	

Fundamental property

The product of the related values is constant.

Example: 10 friends put in €4 each to buy a present for a friend, how much would have to give 8 friends to buy the same present?

PROBLEMS OF INVERSE PROPORTIONALITY

We can solve these problems in two different ways

1. Calculation of the proportionality constant. We calculate the number of units we need.
2. Inverse rule of three.

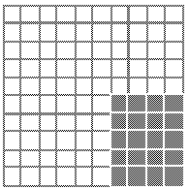
Example: 20 men spend 6 days to clean a village, how long does it take to 30 men?

4. PERCENTAGES

A percentage is a ratio whose denominator is 100.

The shadow part represents 20%.

The total is divided into 100 parts and the shadow part is 20.

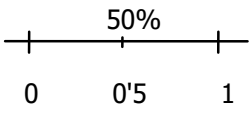


It is denoted by a % or a/100.

To work out the decimal number you only need to divide.

In this case we get the part over one.

50% = 0'5



Por ejemplo:

$54\% = \frac{54}{100}$; $25\% = \frac{25}{100}$; $7\% = \frac{7}{100}$; $120\% = \frac{120}{100}$

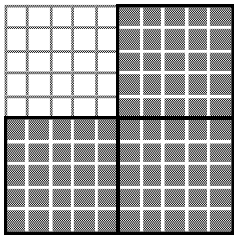
We can use decimal numbers to represent percentages. It is enough to do the division.

$25\% = \frac{25}{100} = 0'25$; $7\% = \frac{7}{100} =$; $120\% =$

To work out the equivalent and irreducible fraction we have to operate in this way:

$$25\% = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

$$75\% = \frac{3}{4} = 75\%$$



RULE TO CALCULATE A PERCENTAGE

We can do a direct rule of three.

Example:

In a class of 30 pupils, 40% of them play football. How many pupils play football in this class?

PERCENTAGE INCREASE AND DECREASE

We consider three questions:

1. The initial quantity. That is the value without the percentage.
2. The final quantity. That is the value with the percentage.
3. The variation index. That is 1 plus or minus the percentage.

Rule of calculation

Final quantity = Initial quantity · Variation index

Initial quantity = Final quantity : Variation index

Example: If we have to pay 21% of V.A.T. for a $250 product; how much tax do we have to pay for it? Calculate it on this way.

VÍDEOS

Ratios:

http://goo.gl/c6LFN7

EXERCISES AND PROBLEMS

(Some exercises are from www.vitutor.com)

1. Ratio and proportion

1. Calculate 'x' to get proportions:

a. $\frac{2}{5}=\frac{7}{x}$; b. $\frac{2}{x}=\frac{x}{50}$; c. $\frac{x}{21}=\frac{7}{3}$

2. Calculate 'x' in the following proportions:

a. $\frac{2}{5}=\frac{9}{x}$; b. $\frac{3}{x}=\frac{x}{27}$;c. $\frac{x}{21}=\frac{7}{4}$

3. Calculate the unknown term in the following proportions:

$\frac{4}{10}=\frac{x}{60}$; $\frac{9}{12}=\frac{12}{x}$; $\frac{8}{32}=\frac{2}{x}$; $\frac{3}{x}=\frac{x}{12}$; $\frac{x}{6}=\frac{24}{x}$

4. Calculate the missing value on these proportions: $a)\frac{3,2}{5,8}=\frac{4,8}{x}$ $b)\frac{4}{x}=\frac{x}{9}$

2. Direct proportionality

5. 4 kilograms of oranges cost 6 euro. How much is a 6 kg bag?

6. Anne buys 5 pounds of potatoes at the market. If 2 pounds cost $0.80, how much does Anne pay?

7. Calculate the height of a building that casts a shadow of 6.5 meters at the same time that a pole with a height of 4.5 m casts a shadow of 0.90 m.

8. To make a cake of 12 portions we have to use: 6 eggs, 150 grams of flour, 100 grams of sugar and a half litre of milk. I am expecting 18 friends at the picnic. How much do I need of everything?

9. We spend 2300 litres of fuel during 20 days for the heating. How many litres will we need for 54 days until the end of winter?

10. Complete the following table:

Nº sweets	6	10	2	8		16	
Price (cents)	90				105		300

11. Answer to the following questions:

a. Give a ratio in direct proportionality to: $\frac{3}{5}$

b. Complete the chart to get a direct proportionality. What is the proportionality constant?

A	2		5	10
B		9	15	

12. A man 1.80 metres tall has got a foot 27 centimetres long. How long is the foot of a boy 80 centimetres tall?

13. There are 20 girls and 10 boys in a classroom. What percentage do they each represent?

14. The sides of a right triangle measure 24 m and 10 m and its hypotenuse is 26 m. What is the length of the sides of a similar triangle whose hypotenuse is 52 m?

15. Three individuals contribute \$5,000, \$7,500 and \$9,000 which are altogether invested into a stock. After a year, the stock has made \$6,450 on top of the capital. If the money is withdrawn, how much will each individual receive if their earnings are directly proportional to the capital originally contributed?

16. The ratio of boys and girls is 4:7 in a classroom.

If the group has 55 students, boys and girls, how many boys and girls are there respectively?

How many girls do we have to add to be in 1:2 proportion?

17. A clock gains 4 minutes every 28 h. How long does it gain every week?

3. Inverse proportionality

18. Answer to the following questions:

a. Give a ratio in direct proportionality to: $\frac{7}{3}$

b. Complete the table to get an ***inverse*** proportionality. What is the proportionality constant?

A	2	3		6
B		4	3	

19. Answer to the following questions:

a. Give a ratio in direct proportionality to: $\frac{3}{5}$

Magnitude A	2	8	**b**	6
Magnitude B	**a**	3	12	**c**

b. Complete the table to get an inverse proportionality. That is, what is the value of a, b and c?

c. What is the proportionality constant?

20. 10 friends spend 4 euros each to buy a gift for a teacher. If they were only 8 friends, how much would each one have to spend?

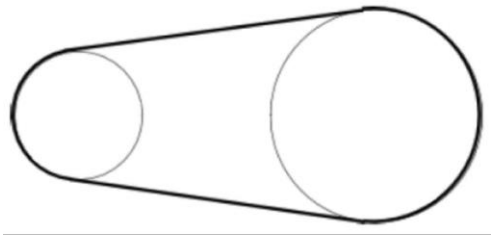
21. Twenty men built a house in six days. How many days do 30 people need to build the house?

22. Four taps fill a water tank in 70 minutes. How long does it take to fill it with seven taps?

23. 35 workers take 16 days to make a fence. How many days will it take 28 workers to raise another?

24. A group of 4 pupils take 8 hours to clean the courtyard. How many students do we need to clean it in two hours and a half?

25. A tap fills a tank in six hours. How long do four taps take to fill the tank?



26. Two workers spend 24 hours to do a job.

a. How long do 3 workers spend on it?

b. How many workers are necessary to do this in 6 hours?

27. Two gears are connected by a chain. The first has a radius of 25 cm and the second, 75 cm. When the first has reached 300 turns, how many turns will the second have completed?

28. John won an award and he thinks about buying toys for 40 needy children. The shop assistant is going to reduce the price from 14 euros to 12 euros for each toy. How many children could he aid now?

4. Percentages

29. A trader sells an item for 20€ plus 16 % of VAT. How much is the final price?

30. A pair of trouser which is €42 has a 20% discount in a sale. How much is its new price?

31. A bicycle is 236 euros. It has a 7% tax. How much does the State receive from this sale?

32. We get a 180-euro discount from the 1500 euros that a computer costs. What percentage of discount does it have?

33. The price of a toy is 23.5€ with tax included (16%). What is the price without V.A.T. (value added tax)?

34. In a school, 25% of students play soccer, 40% play basketball and the rest swim. If the school has 800 students, how many students practice each sport?

35. An item cost 10 € last year and this year it costs 11.50 €. What percentage has the price increased?

36. Of the 800 students at a school, 600 attend a field trip. What is percentage of students who did not remain at the school?

37. When purchasing a vehicle priced at $8,800, the sales clerk awards a discount of 7.5%. How much will the final cost of the vehicle be?

38. The price of a computer is $1,200. How much will a customer need to pay in total if the tax is 16%?

39. If a monitor is priced at $450 and the store manager awards a discount of 8%, how much will the costumer pay?

40. An item is sold for 15% more than its original price. The retailer sells it for $80, determine the original price.

41. What is the price that a retailer has to sell an item acquired for $180 in order to make a profit of 10%?

42. What is the sale price that a retailer has to put on an item originally priced at $280 to reduce it by 12% for an upcoming sale?

43. An object is sold during a "20% off" sale. Find the sale price of that item if its original price was $150.

44. Four litres of air contains 3.6 litres of water vapour. What is the percentage of air humidity?

45. A disease killed a 22% of pigs in a farm. Today there are 273, how many pigs were there before the epidemic?

46. Laura has used 10 paper sheets of her notebook during the first 15 days of class. She is wondering if she will have enough space to finish the school year which is 90 days long knowing her notebook is 80 pages.